Quantifying Cryptocurrency Unpredictability: A Comprehensive Study of Complexity and Forecasting

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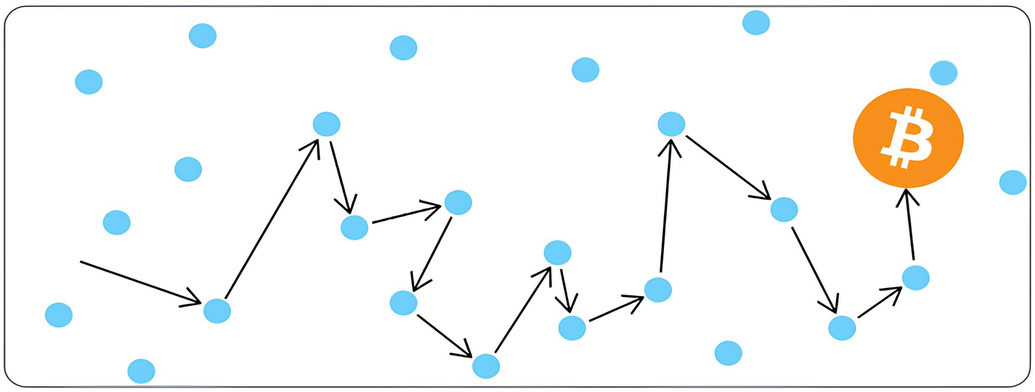

Figure 1: Cryptocurrency as Brownian Motion

Abstract

This paper offers a thorough examination of the univariate predictability in cryptocurrency time-series. By exploiting a combination of complexity measure and model predictions we explore the cryptocurrencies time-series forecasting task focusing on the exchange rate in USD of Litecoin, Binance Coin, Bitcoin, Ethereum, and XRP. On one hand, to assess the complexity and the randomness of these time-series, a comparative analysis has been performed using Brownian and colored noises as a benchmark. The results obtained from the Complexity-Entropy causality plane and power density spectrum analysis reveal that cryptocurrency time-series exhibit characteristics closely resembling those of Brownian noise when analyzed in a univariate context. On the other hand, the application of a wide range of statistical, machine and deep learning models for time-series forecasting demonstrates the low predictability of cryptocurrencies. Notably, our analysis reveals that simpler models such as Naive models consistently outperform the more complex machine and deep learning ones in terms of forecasting accuracy across different forecast horizons and time windows. The combined study of complexity and forecasting accuracies highlights the difficulty of predicting the cryptocurrency market. These findings provide valuable insights into the inherent characteristics of the cryptocurrency data and highlight the need to reassess the challenges associated with predicting cryptocurrency's price movements.

CCS Concepts

• **Applied computing → Mathematics and statistics**; **Economics**; • **Computing methodologies → Neural networks**; • **Mathematics of computing → Time series analysis**.

Keywords

Time series Forecasting, Time series Complexity, Time series Entropy, Cryptocurrencies, Preemptive Analysis, Machine Learning, Deep Learning, Naive Models

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1 Introduction

Cryptocurrencies have emerged as a significant asset in the global financial system, offering a new paradigm for digital transactions [19]. Their decentralized nature, coupled with their potential for high returns, has attracted the attention of investors, researchers, and financial institutions alike [4]. The cryptocurrency market is influenced by several key factors. Market sentiment and speculation play a significant role, with prices being highly sensitive to social media, news events, celebrity endorsements, and public perceptions. These speculative behaviors cause what are perceived as rapid and unpredictable price changes, making it difficult to distinguish between genuine value shifts and short-term market reactions [18]. Additionally, the evolving and inconsistent regulatory environment is another influential factor. Announcements of new regulations, enforcement actions, or changes in legal status can lead to sudden and volatile price movements, creating an unstable investment landscape. This volatile and complex behavior of cryptocurrencies presents unique challenges for traditional financial analysis and forecasting methods [2].

This study analyzes on an extended time period five prominent cryptocurrencies' exchange rates in USD: Litecoin (LTC-USD), Binance Coin (BNB-USD), Bitcoin (BTC-USD), XRP (XRP-USD), and Ethereum (ETH-USD). We aim to provide a comprehensive understanding of cryptocurrency dynamical features and put under scrutiny their predictability in a univariate context. To this aim, this study leverages complexity measures such as the Permutation Entropy [3] and the CH-plane [21], and statistical, machine and deep learning models for time-series analysis, ranging from simple Naive models and ARIMA [14] up to more complex models such as XGBModels [8] and NBEATS [20]. This approach combines advanced statistical complexity methods with state-of-the-art machine learning techniques to capture the intricate patterns and potential predictability in cryptocurrency markets [16]. Our methodology includes the analysis of cryptocurrencies through the Complexity-Entropy causality plane (CH-plane) [21] and power density spectrum. These tools allow us to draw parallels with Brownian motion, a well-known stochastic process that describes the random motion of particles suspended in a fluid. Interestingly, Brownian motion is often used as a benchmark for random behavior in financial time-series and has a long tradition in the modelling of the stock market [1]. By comparing the characteristics of cryptocurrency time-series to Brownian motion, this study aims to gain insights into the efficiency and randomness of these markets, ultimately addressing the question of whether cryptocurrencies exhibit predictable patterns or are essentially pure noise. Furthermore, we employ a range of forecasting models, from simple statistical approaches to sophisticated machine and deep learning algorithms, to assess the predictability of cryptocurrency price movements [9]. This comparative analysis not only evaluates the performance of different forecasting techniques but also provides insights into the inherent predictability of cryptocurrency markets. Our research emphasizes the potential effectiveness of simpler forecasting methods under certain conditions, challenging the hypothesis that more sophisticated models always yield better results in the context of cryptocurrency time-series forecasting [26]. By combining complexity analysis with predictive modeling, this study aims to provide a comprehensive understanding of cryptocurrency dynamical behavior and predictability.

2 Related literature

Permutation entropy and the Complexity-Entropy (CH) plane have been widely used to characterize the complexity and predictability of time-series data. The concept of permutation entropy as a measure of the degree of randomness in a time-series was introduced in [3]. This method has gained popularity due to its simplicity and effectiveness in quantifying the temporal structure of complex systems. The CH-plane, proposed in [21], plots permutation entropy against the statistical Jensen-Shannon complexity measure $C_{JS}[P]$, is a functional of the probability distribution P associated with the time series [3], allowing for the classification of different dynamical regimes. This approach has been successfully applied to various fields, including financial markets [28].

The relationship between the CH-plane and the predictability of cryptocurrency time-series has been studied in several works. Notably, in [23] the authors analyzed the entropy and statistical complexity of Bitcoin and Ethereum time-series, suggesting that the price dynamics are largely driven by noise. This finding aligns with the efficient market hypothesis, which posits that asset prices fully reflect all available information, making them inherently unpredictable [29]. Similarly, [23] applied the CH-plane methodology to map cryptocurrencies and found that their behavior varies widely within the plane, with price dynamics ranging from stochastic to more structured. This work observed that cryptocurrencies with high market capitalization tend to be more complex and less entropic than those with very low market capitalization, suggesting that major cryptocurrencies are less market efficient. While these studies have employed complexity measures and the CH-plane to assess the nature of cryptocurrency time-series, they are based on much shorter time windows and have primarily focused on characterizing the dynamics rather than directly evaluating predictability.

Despite the challenges posed by the noisy nature of cryptocurrency time-series, some studies have reported successful predictions using various machine learning (ML) and deep learning (DL) models. However, it is crucial to note that most of these models were not compared with naive forecasting methods, or they introduced past and future covariates in the forecasting task, potentially inflating their perceived effectiveness. For example [5] employed a range of ML models to predict cryptocurrency returns, reporting promising results. However, the study did not include a comparison with naive forecasting methods. The study [7] used GARCH-type models to forecast cryptocurrency volatility, incorporating exogenous variables, which may have contributed to the model's performance. In [15] the authors applied a DL-based approach for cryptocurrency price prediction, reporting high accuracy. However, this study did not include a comparison with simpler forecasting methods. Similarly, [17] used Recurrent Neural Networks and Long Short-Term Memory networks for Bitcoin price prediction, but also lacked comparisons with baseline models. The authors of [22] integrated additional market indicators and sentiment analysis in their forecasting models, potentially improving predictions but deviating from a purely univariate approach. More recently, [16] investigated the predictability of cryptocurrency trading volume using support vector regression (SVR) with different kernels. They found that SVR with radial basis function kernel outperformed other models for next-day trading volume prediction, while SVR with polynomial kernel was superior for next-week predictions. These studies highlight the importance of a careful model evaluation and comparison with simple benchmarks to assess the true predictive power of complex models in cryptocurrency forecasting. Our study aims to address this gap by providing a comprehensive comparison of various forecasting methods, including naive models, in a purely univariate context.

3 Methodology

3.1 Data Collection

The daily pricing data of the cryptocurrencies used in this study are collected from the Yahoo Finance database. Each cryptocurrency time-series y ranges from 2020-07-03 to 2023-12-21, and no data preprocessing was applied. The data was split into two subsets, i.e., a training series y_{train} and a test series y_{target}. The split date index is 2023-07-04, meaning that the last 180 data points represent y_{target}.

To comprehensively evaluate the forecasting models (statistical, ML, and DL) and conduct the complexity analysis, we adopted a multiple timescale approach. This allows for a more in-depth understanding of cryptocurrency dynamics across various time horizons. Three different training time-window lengths (t_w) were applied on y_{train}, i.e., (i) t_w = 3 years, (ii) t_w = 1 year and (iii) t_w = 6 months. Using these three different t_w values allows us to identify whether there are long-term, mid-term, or short-term patterns, respectively, within the cryptocurrency time-series data. The y_{target} is the same for each considered time window t_w.

A visual summary of the cryptocurrency univariate time-series and scenarios considered in this paper is shown in Fig. 2. The considered scenarios are:

(1) t_w = 3 years,
$y_{train} = \{y(t)\ \forall\ 2020\text{-}07\text{-}03 \leq t \leq 2023\text{-}07\text{-}04\}$
(2) t_w = 1 year,
$y_{train} = \{y(t)\ \forall\ 2022\text{-}07\text{-}03 \leq t \leq 2023\text{-}07\text{-}04\}$
(3) t_w = 6 months,
$y_{train} = \{y(t)\ \forall\ 2023\text{-}01\text{-}04 \leq t \leq 2023\text{-}07\text{-}04\}$

3.2 Complexity Measures

In order to evaluate the statistical complexity and the level of disorder and unpredictability of each cryptocurrency this study relies on the Bandt and Pompe *permutation entropy* [3] (PE) and the *intensive statistical complexity measure* $C_{JS}[P]$ proposed in [21], situating each cryptocurrency in the *Complexity-Entropy* (CH) plane [21].

The permutation entropy quantifies the degree of randomness inherent in a process; the lower the entropy, the higher the predictability of the process. Focusing on the relative ordering of the time-series values, the permutation entropy takes into account the temporal causality within the series. For a time-series of length n, ordinal patterns of user-defined positive integer size $d : n \geq 5d!$ [23] are created. Then, the relative frequencies of these patterns are calculated to form a probability distribution. The permutation entropy is defined as the Shannon entropy of this distribution. It ranges from 0, which represents complete predictability, to $log(d!)$, which represents maximum randomness. In this study, we set $d = 5$ for t_w = 3 years, and $d = 4$ for t_w = 1 year and t_w = 6 months.

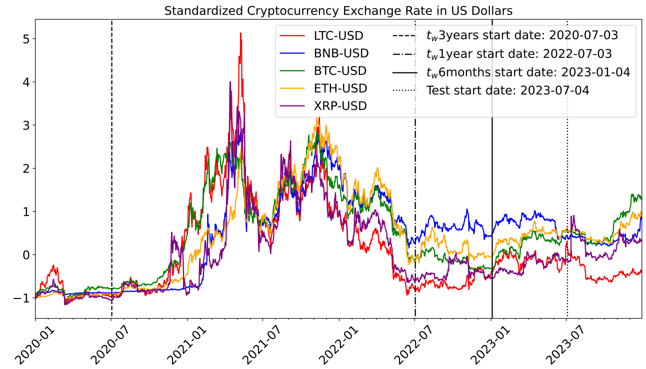


Figure 2: Overview of the Cryptocurrency time-series analyzed in this paper. The different starting dates relative to the three chosen time windows t_w are shown. The cryptocurrency time-series have been standardized for clarity of the plot.

Furthermore, we employ the Jensen–Shannon complexity measure $C_{JS}[P]$ to quantify the complexity of the underlying probability distribution P of the time-series. This measure integrates the concepts of permutation entropy and Jensen–Shannon divergence (D_{JS}), enabling a comprehensive capture of both the uncertainty and the structure of the distribution. The calculation of $C_{JS}[P]$ involves a two-step process: first, determining the D_{JS} between the probability distribution P and a uniform distribution, and then multiplying this divergence by the entropy of P. This approach provides valuable insights into the system's complexity by simultaneously accounting for two critical aspects: *diversity*, which represents the amount of uncertainty or unpredictability in the distribution, and *distinctiveness*, which quantifies the degree to which privileged fluctuations exist among those accessible to the system [23].

By calculating both quantities, valuable insight can be gained regarding the distribution and the degree of correlations of time-series patterns, thereby reflecting the interplay between order and disorder. Plotting the PE and the $C_{JS}[P]$ of a time-series in the CH-plane, one can distinguish between different dynamical behaviors: deterministic time-series are typically characterized by low entropy and high complexity, while purely random processes such as Brownian motion by high entropy and low complexity.

Furthermore, we employ the *permutation Jensen-Shannon distance* (PJSD) [27] to quantify the degree of similarity between each cryptocurrency time-series and various colored noises. This measure combines the concepts of Jensen-Shannon divergence (D_{JS}) and Permutation Entropy (PE) to provide a robust metric for comparing time-series. The PJSD is calculated as $\sqrt{D_{JS}(P,Q)}$ where P and Q are the ordinal probability distributions associated with the two time-series under analysis. In our case, P represents the distribution of a cryptocurrency time-series, while Q represents the distribution of a specific colored noise. By computing the PJSD between each cryptocurrency and various colored noises, we can precisely quantify how closely the dynamics of cryptocurrency markets resemble different types of random processes.

Finally, we evaluate the cryptocurrency time-series in the frequency domain through the *Power Spectral Density* (PSD). The PSD plot shows how the power of a signal is distributed over frequencies. It provides key insights into the periodicities, dominant frequencies, and scaling behaviors of the time-series. In particular, the power spectral density $S(f)$ of colored noises follows a power law distribution as a function of the frequency f: $S(f) \propto 1/f^{\alpha}$, where α is the power-law exponent characterising each colored noise. For instance, a lower absolute value of α indicates that the time-series is close to white noise ($\alpha = 0$ being its characteristic exponent), while a higher absolute value of α indicates that the time-series reveals more structured patterns.

3.3 Statistical, Machine and Deep Learning Models

In this study, we employ a wide range of models for time-series forecasting, including statistical models, ML models, and DL models. Comparing the performance of different types of models enables us to gain insights into the underlying structure of the time-series, the complexity, the predictability, and the nature of the information present in the data. By comparing the performance of the different model types, we can infer whether the time-series exhibit identifiable patterns that can be learned and predicted, or if it is inherently noisy and lacks significant predictable signals. This approach enables a comprehensive assessment of cryptocurrency market dynamics and of the efficacy of various forecasting techniques in this financial domain.

The complete list of models used in our analysis is shown in Table 1. The models and the back-testing procedure are implemented using the Darts [12] Python library.

Table 1: List of Models by Category

Class of Models	Models
Statistical	NaiveDrift [14], NaiveSeasonal [14], ARIMA [14], Exponential Smoothing (ETS) [14] Complex Exponential Smoothing (CES) [24] TBATS [10], Prophet [25]
Machine Learning	RandomForest [6], XGBModel [8]
Deep Learning	VanillaRNN [11], LSTM [13], NBEATS [20]

The back-testing procedure trains a model M with a specified forecast horizon f_h and time window t_w. It begins by training M on the initial y_{train} dataset, constrained to the defined time window t_w. The model then generates a forecast spanning f_h time steps. Following this, the procedure incrementally expands y_{train} by incorporating one sample from y_{target}. This process repeats iteratively, continuously updating the training set and producing new forecasts. This rolling window approach enables a comprehensive evaluation of the model's predictive performance across various temporal segments of the data. More specifically, at time t, the forecast is obtained as:

$$\tilde{y}(t) = M(y(t - fh), y(t - fh - 1), \ldots, y(t - fh - t_w)).$$

The forecasting metric used for comparing the models accuracy is the Mean Absolute Percentage Error (MAPE), defined as:

$$\text{MAPE}\ (\tilde{y}, y_{target}) = \frac{1}{T}\sum_{t=1}^{T}\left|\frac{\tilde{y}(t) - y_{target}(t)}{y_{target}(t)}\right| \times 100$$

where $\tilde{y}$ corresponds to the forecast of a given model, $t = 1$ is the first forecast index and T is the size of y_{target}. The MAPE metric is evaluated over the entire y_{target} by using only the point predicted at the given forecast horizon f_h. This approach allows for a focused evaluation of the models' predictive accuracy at specific future time points, as shorter-forecast points do not influence the metric computation.

4 Results

4.1 Complexity measures

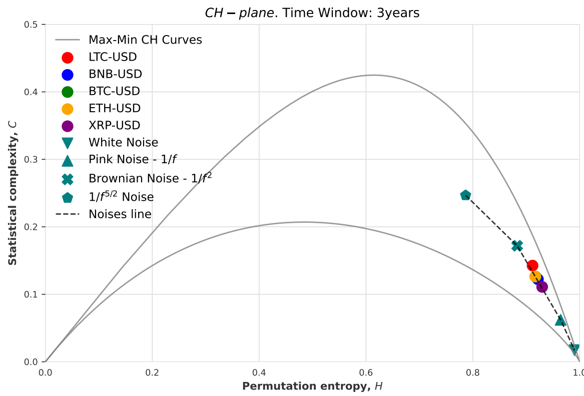
The CH-plane plots depicted in Fig. 3a, 3b and 3c show that all cryptocurrencies are situated in the lower right corner, indicating high permutation entropy and low statistical complexity. This positioning suggests that their statistical properties closely resemble those of different types of noise, i.e., colored noises such as white, $1/f$ (pink), $1/f^2$ (Brownian), and $1/f^{5/2}$ noises, which are represented in Fig. 3 along the dashed "noises line". Indeed, the cryptocurrency time-series lie precisely on the noises line.

The high permutation entropy values imply a significant degree of randomness and unpredictability, while the relatively low statistical complexity values indicate minimal underlying structure. Moreover, as the time window shortens, i.e., from $t_w = 3$ years to $t_w = 6$ months, the statistical characteristics of the cryptocurrency time-series increasingly align with those of white noise. This behaviour suggests a growing level of randomness and unpredictability, and the lack of meaningful patterns in cryptocurrency time-series particularly over short-term periods.

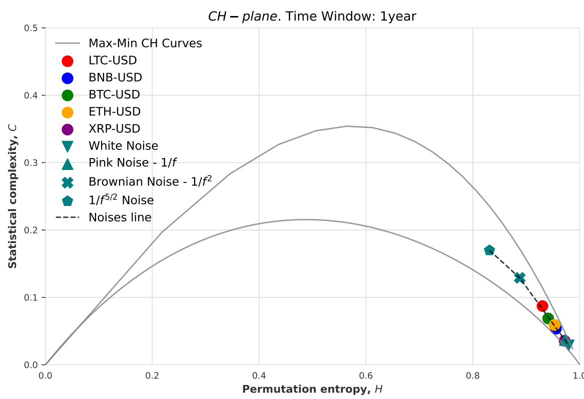
To quantify these similarities more precisely, we computed the PJSD between each cryptocurrency and various types of colored noise over the full available time window 2020-07-03 to 2023-12-21. Results are shown in Table 2 and provide a clear comparative analysis of which type of noise each cryptocurrency most closely resembles. The PJSD values indicate that, for most cryptocurrencies, the minimum distance is observed with respect to Brownian noise. However, an exception is noted with XRP-USD, which shows a higher similarity to pink noise.

Table 2: Permutation Jehnsenn-Shannon distances between Cryptocurrencies and colored noises over the full available time window 2020-07-03 to 2023-12-21. The minimum distance is observed with Brownian noise for most cryptocurrencies. XRP-USD represents an exception showing a higher similarity to pink noise.

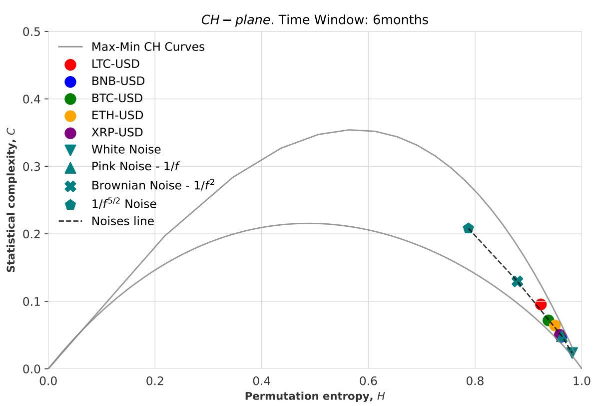
	White	Pink - $1/f$	Brownian - $1/f^2$	$1/f^{5/2}$	$1/f^3$
LTC-USD	0.368	0.215	**0.198**	0.334	0.539
BNB-USD	0.358	0.212	**0.209**	0.353	0.552
BTC-USD	0.355	0.211	**0.194**	0.338	0.530
ETH-USD	0.356	0.202	**0.198**	0.339	0.536
XRP-USD	0.351	**0.214**	0.230	0.374	0.566



(a) CH-plane: time window t_w = 3 years



(b) CH-plane: time window t_w = 1 year



(c) CH-plane: time window t_w = 6 months

Figure 3: Complexity Entropy causality plane (CH-plane) of cryptocurrencies time-series (LTC-USD, BNB-USD, BTC-USD, ETH-USD, XRP-USD) compared to different types of noises: white, $1/f$ (pink), $1/f^2$ (Brownian), and $1/f^{5/2}$, computed using different time windows settings. All the cryptocurrencies lie on the dashed line characterizing the colored noises. As the time window shortens, the cryptocurrency data increasingly align with the position of white noise.

The PSD curves shown in Fig. 4 are computed by using the whole available data time range, i.e., from 2020-07-03 to 2023-12-21. The plot corroborates the similarity of the cryptocurrency time-series to $1/f^2$ (Brownian) noise whose exponent is represented by the full black reference line. In addition, the analysis was extended to shorter time windows, i.e. $t_w = 3$ years, $t_w = 1$ year, and $t_w = 6$ months, as well. Interestingly, the spectral characteristics of the cryptocurrencies remain consistent, i.e., the PSD consistently follows the power law decay exponent of the Brownian motion, regardless of the time window. The plot of the PSD for the other time windows is not reported here for brevity.

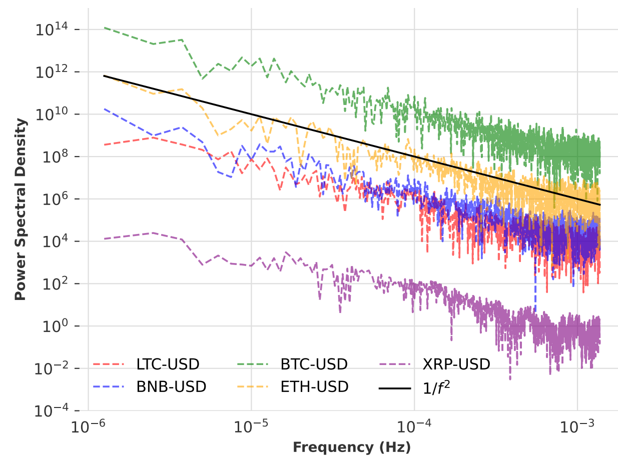


Figure 4: Power Spectral Density (PSD) plots of the 5 cryptocurrency univariate time-series (LTC-USD, BNB-USD, BTC-USD, ETH-USD, XRP-USD) computed using the whole available data time range (from 2020-07-03 to 2023-12-21). The PSD plots shows that cryptocurrency time-series follow a power-law distribution comparable to Brownian noise, whose exponent is indicated by the full black reference line.

4.2 Model performances

By examining Tables 3a, 3b and 3c, several observations can be made regarding the prediction accuracy of the considered models across cryptocurrencies, time windows t_w, and forecast horizons f_h. Notably, as shown in Table 3, one of our core results is that statistical approaches consistently outperform their ML and DL counterparts.

Focusing on the statistical approaches, models like AutoARIMA, AutoETS, AutoCES, and TBATS did not significantly over-perform the much simpler Naive models. In fact, despite the greater sophistication and theoretical guarantees of such models, the fitting procedures of these advanced models resulted in average performances that are not statistically distinguishable from those achieved by the Naive models. The aggregated view in Table 4, where the mean MAPE over all cryptocurrencies and all t_w is reported, together with the corresponding standard deviations, substantiates this result.

In contrast, the Prophet model resulted in much worse performances compared to the other statistical methods, and in some

Table 3: MAPE results for statistical and ML/DL models across five cryptocurrencies (LTC-USD, BNB-USD, BTC-USD, ETH-USD, XRP-USD) for (a) 3-year, (b) 1-year, and (c) 6-month time windows, with forecast horizons of 1, 7, and 30 days. A key finding is that statistical approaches consistently outperform ML and DL models, with the exception of NBEATS, which shows competitive but not consistently superior performance. Interestingly, sophisticated statistical models like AutoARIMA, AutoETS, AutoCES, and TBATS did not significantly outperform simpler Naive models. DL models, despite their reputation for modeling complex sequences, face challenges in cryptocurrency forecasting. As the time window shrinks, prediction errors for VanillaRNN and LSTM models increase across all forecast horizons. Similarly, traditional ML algorithms like RandomForest and XGBModel struggle to discern patterns amid the inherent noise and volatility.

(a)

Time Window	3 years														
Dataset	LTC-USD			BNB-USD			BTC-USD			ETH-USD			XRP-USD		
Forecast Horizon	1	7	30	1	7	30	1	7	30	1	7	30	1	7	30
NaiveDrift	1.914	5.361	12.562	1.415	3.808	7.815	1.298	3.617	**11.012**	1.452	3.946	9.203	2.110	6.600	16.589
NaiveSeasonal	**1.911**	5.298	12.436	1.419	3.759	7.049	1.296	3.619	11.307	1.448	3.886	**9.011**	2.105	6.517	16.276
AutoARIMA	1.929	5.318	12.444	1.437	3.770	7.055	1.296	3.619	11.307	**1.447**	3.875	9.011	2.093	**6.5**	**16.264**
AutoETS	1.933	5.715	14.884	1.433	4.065	10.249	**1.292**	**3.579**	11.071	1.461	3.968	9.439	2.131	6.983	18.397
AutoCES	1.938	**5.233**	12.766	**1.408**	**3.735**	**7.023**	1.310	3.652	11.662	1.452	**3.843**	9.096	**2.07**	6.529	17.290
TBATS	1.912	5.294	**12.429**	1.410	3.832	7.685	1.293	3.618	11.310	1.447	3.880	9.011	2.094	6.506	16.267
Prophet	22.301	27.390	42.971	11.929	14.062	18.253	10.140	13.130	23.244	12.247	15.457	24.848	15.492	18.903	27.875
RandomForest	2.202	8.234	18.755	1.651	4.974	25.132	1.675	6.531	11.728	1.694	5.117	10.593	2.364	9.501	30.308
XGBModel	2.707	10.355	19.039	1.985	5.483	24.539	1.902	6.546	13.549	1.850	5.577	12.534	2.715	12.416	37.773
VanillaRNN	2.064	7.209	19.112	2.967	6.772	8.973	99.091	99.102	99.120	84.672	84.746	85.049	2.237	7.922	29.138
LSTM	2.195	7.461	19.425	2.564	7.541	9.226	99.765	99.789	99.794	93.190	93.211	93.349	2.333	8.359	31.029
NBEATS	2.717	7.730	16.043	2.023	5.680	23.357	1.875	4.801	15.742	2.006	5.512	12.853	3.393	13.202	32.761

(b)

Time Window	1 year														
Dataset	LTC-USD			BNB-USD			BTC-USD			ETH-USD			XRP-USD		
Forecast Horizon	1	7	30	1	7	30	1	7	30	1	7	30	1	7	30
NaiveDrift	1.919	5.458	12.995	1.420	3.779	7.277	1.299	3.627	11.077	1.455	3.967	9.390	2.117	6.703	17.179
NaiveSeasonal	**1.911**	**5.298**	12.436	1.419	3.759	7.049	1.296	3.619	11.307	1.448	3.886	**9.011**	2.105	**6.517**	16.276
AutoARIMA	1.911	5.298	12.436	1.404	3.741	7.035	1.296	3.619	11.307	1.449	3.877	9.011	2.103	6.527	16.278
AutoETS	1.911	5.298	12.436	**1.402**	**3.735**	**7.032**	1.296	3.619	11.307	1.447	**3.875**	9.012	2.079	6.555	16.243
AutoCES	1.945	5.335	13.348	1.412	3.750	7.104	1.315	3.612	11.230	1.457	3.881	9.150	2.104	6.743	17.514
TBATS	1.914	5.306	**12.271**	1.429	4.067	9.189	**1.293**	**3.595**	**10.605**	**1.44**	3.903	9.226	**2.071**	6.606	17.082
Prophet	11.709	14.065	22.853	6.931	8.420	15.934	7.437	9.022	15.100	7.054	8.407	13.501	12.627	14.862	21.915
RandomForest	2.080	6.878	19.199	1.685	4.866	17.374	1.693	5.955	12.300	1.773	4.597	10.449	2.699	9.355	20.851
XGBModel	2.499	8.870	20.802	1.875	5.475	18.333	2.029	6.914	11.978	1.963	5.012	11.227	3.082	10.037	24.186
VanillaRNN	8.156	11.960	12.618	56.079	56.673	58.750	99.658	99.674	99.691	94.179	94.339	94.640	2.219	6.955	**16.117**
LSTM	8.217	10.302	13.364	77.060	78.361	79.712	99.912	99.929	99.936	98.403	98.613	98.772	2.127	6.824	17.304
NBEATS	2.573	7.270	16.216	2.015	4.593	9.293	1.744	4.741	13.307	1.835	4.727	11.594	3.283	9.125	22.532

(c)

Time Window	6 months														
Dataset	LTC-USD			BNB-USD			BTC-USD			ETH-USD			XRP-USD		
Forecast Horizon	1	7	30	1	7	30	1	7	30	1	7	30	1	7	30
NaiveDrift	1.922	5.454	13.676	1.425	3.803	7.433	1.315	3.787	11.247	1.463	4.062	9.966	2.130	6.881	18.209
NaiveSeasonal	1.911	5.298	12.436	1.419	3.759	7.049	**1.296**	3.619	11.307	**1.448**	3.886	9.011	2.105	**6.517**	16.276
AutoARIMA	**1.907**	**5.205**	**11.907**	1.401	3.759	8.460	1.296	3.619	11.307	1.456	3.871	9.014	2.115	6.565	16.261
AutoETS	1.911	5.298	12.435	**1.4**	**3.742**	**7.043**	1.298	**3.617**	11.157	1.449	**3.869**	**9.007**	**2.073**	6.578	16.230
AutoCES	1.948	5.359	13.823	1.414	3.763	7.291	1.333	3.765	11.306	1.472	3.992	9.788	2.138	6.990	18.763
TBATS	2.005	6.319	14.243	1.471	3.809	8.267	1.315	3.784	**10.292**	1.487	4.378	10.124	2.088	6.631	16.286
Prophet	8.903	11.634	23.436	5.173	6.805	12.722	5.532	7.451	16.171	4.631	6.163	12.856	10.751	13.959	25.909
RandomForest	2.417	8.036	18.194	1.652	4.149	14.504	1.655	6.077	12.362	1.826	4.498	9.907	2.728	9.237	20.008
XGBModel	2.689	9.011	20.523	1.873	5.081	15.371	1.803	6.350	11.773	2.082	4.771	10.499	3.947	9.819	24.050
VanillaRNN	25.377	26.717	31.597	73.680	74.399	76.592	99.801	99.820	99.838	96.590	96.848	97.161	2.279	6.761	**15.244**
LSTM	27.644	28.832	33.609	86.521	87.064	88.554	99.949	99.966	99.975	99.096	99.380	99.550	2.408	6.637	18.355
NBEATS	2.736	6.877	15.221	1.782	4.293	9.635	1.847	5.061	12.649	1.984	4.599	10.543	3.854	10.144	21.714

Table 4: Aggregated view of mean MAPE ± standard deviation across all cryptocurrencies and time windows corresponding to tables (a), (b) and (c) in Table 3, for statistical, machine learning (ML), and deep learning (DL) models. This view illustrates that while statistical approaches generally outperform ML and DL methods, the complex statistical techniques (AutoARIMA, AutoETS, AutoCES, and TBATS) show no statistically significant improvement over Naive models, as evidenced by the overlapping standard deviations.

Forecast Horizon	1	7	30
NaiveDrift	1.644 ± 0.328	4.723 ± 1.227	11.709 ± 3.496
NaiveSeasonal	1.636 ± 0.325	4.616 ± 1.166	**11.216 ± 3.251**
AutoARIMA	1.636 ± 0.326	**4.611 ± 1.17**	11.273 ± 3.128
AutoETS	**1.634 ± 0.324**	4.7 ± 1.245	11.729 ± 3.415
AutoCES	1.648 ± 0.326	4.679 ± 1.244	11.81 ± 3.807
TBATS	1.645 ± 0.321	4.768 ± 1.21	11.619 ± 3.064
Prophet	10.19 ± 4.614	12.649 ± 5.532	21.173 ± 7.865
RandomForest	1.986 ± 0.397	6.534 ± 1.903	16.778 ± 5.903
XGBModel	2.333 ± 0.606	7.448 ± 2.423	18.412 ± 7.345
VanillaRNN	49.937 ± 43.905	51.993 ± 41.825	56.243 ± 37.961
LSTM	53.426 ± 45.967	55.485 ± 43.899	60.13 ± 39.327
NBEATS	2.378 ± 0.679	6.557 ± 2.567	16.231 ± 6.4

cases, even compared to ML and DL models. This discrepancy can be attributed to the specific use-cases Prophet is designed to address, characterized by strong seasonal effects and holidays. Hence, the high volatility and the absence of regular seasonal patterns in the cryptocurrency markets present a challenging environment for Prophet's underlying assumptions and mechanisms.

DL models, renowned for their ability to model complex sequences and capture long-term dependencies, are often seen as holding the potential of superior performance in time series forecasting. However, their application to univariate cryptocurrency time series reveals inherent challenges. These models require substantial amounts of data to be trained effectively and are prone to overfitting, especially when compared to the noise-dominated nature of cryptocurrency data. These limitations become evident when examining the MAPE across decreasing time windows, as can be seen in Table 3. The results clearly indicate that as the time window t_w shrinks, the prediction error of VanillaRNN and LSTM models increases, irrespective of the forecast horizon f_h. The NBEATS model stands out as a notable exception. However, while it delivers competitive prediction accuracy, its performance does not statistically match the top statistical models. This unexpected result underscores a critical insight: even advanced DL architectures may struggle in domains characterized by high volatility and noise.

Akin to deep learning models, traditional ML algorithms like RandomForest and XGBModel - often acclaimed for their capacity to capture intricate non-linear relationships and maintain robust generalization - underperform in univariate cryptocurrency forecasting. These models also encounter difficulty in discerning significant patterns amid the stochastic fluctuations and inherent noise of cryptocurrency markets.

5 Discussion and Conclusion

Our study reveals that univariate forecasting of cryptocurrencies is essentially comparable to pure noise forecasting. Simpler statistical models are consistently comparable or outperform more complex ML and DL models across various forecast horizons and time windows in an extended time range from 2020-07-03 to 2023-12-21, for five prominent cryptocurrencies, i.e. LTC-USD, BNB-USD, BTC-USD, ETH-USD and XRP-USD. Complexity analysis using the CH-plane and the Power Spectral Density (PSD) highlights the noisy nature of cryptocurrency time-series, revealing high entropy, low complexity, and PSD power law exponents comparable with those of Brownian motion. These findings collectively demonstrate the inherent stochastic nature of cryptocurrencies and the varying degrees of noise-like behavior they exhibit over different time scales. These insights challenge the presence of predictable patterns in cryptocurrency markets and suggest that their apparent complexity may be largely attributed to noise. The resemblance to Brownian motion implies that forecasting future prices based solely on historical data may be unfeasible. Likewise, the similarity to white noise over shorter periods points to increased randomness and potential challenges even in short-term forecasting.

This study challenges the conventional wisdom that increased model complexity guarantees better performance. The inherent unpredictability and rapid evolution of cryptocurrency markets pose significant hurdles for deep learning and machine learning models. These models, often acclaimed for their sophisticated designs, may not consistently deliver superior performance across all contexts. This aligns with the findings in [15], which observed that simple forecasting methods can outperform more complex ones in cryptocurrency markets. Our study invites researchers and practitioners to reconsider their approach to model selection, emphasizing the value of simple models for what concerns the ability to handle noise and volatility.

However, it's important to note that incorporating additional covariates can significantly improve the forecasting accuracy of cryptocurrency models. These covariates can be categorized into past covariates, such as technical indicators and correlated time-series data, and future covariates, including scheduled events and macroeconomic forecasts. Past covariates may help mitigate noise and complexity in the models, while future covariates can assist in anticipating external influences on cryptocurrency markets. While past covariates are generally easier to obtain and incorporate, future covariates present some challenges. They are often difficult to retrieve directly, and when forecasted rather than known with certainty, they can introduce additional uncertainty into the model. This observation is consistent with the findings of [22], which demonstrated improved forecasting performance by integrating additional market indicators and sentiment analysis. Similarly, [7] showed that incorporating exogenous variables can enhance cryptocurrency volatility forecasting.

In conclusion, our study highlights the importance of balancing model complexity with the inherent noise and unpredictability in cryptocurrency markets. While more sophisticated models may offer potential benefits, the effectiveness of simpler models should not be underestimated. Future research should focus on identifying the most relevant and impactful covariates for cryptocurrency forecasting, as well as developing methods to effectively incorporate future covariates without introducing excessive uncertainty.

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References

[1] W Farida Agustini, Ika Restu Affianti, and Endah RM Putri. 2018. Stock price prediction using geometric Brownian motion. *Journal of Physics: Conference Series* 974, 1 (mar 2018), 012047. https://doi.org/10.1088/1742-6596/974/1/012047

[2] Erdinc Akyildirim, Ahmet Goncu, and Ahmet Sensoy. 2021. Prediction of cryptocurrency returns using machine learning. *Annals of Operations Research* 297, 1 (01 Feb 2021), 3–36. https://doi.org/10.1007/s10479-020-03575-y

[3] Christoph Bandt and Bernd Pompe. 2002. Permutation entropy: a natural complexity measure for time series. *Physical review letters* 88, 17 (2002), 174102.

[4] Dirk G. Baur, KiHoon Hong, and Adrian D. Lee. 2018. Bitcoin: Medium of exchange or speculative assets? *Journal of International Financial Markets, Institutions and Money* 54 (2018), 177–189. https://doi.org/10.1016/j.intfin.2017.12.004

[5] Ahmed Bouteska, Mohammad Zoynul Abedin, Petr Hajek, and Kunpeng Yuan. 2024. Cryptocurrency price forecasting – A comparative analysis of ensemble learning and deep learning methods. *International Review of Financial Analysis* 92 (2024), 103055. https://doi.org/10.1016/j.irfa.2023.103055

[6] Leo Breiman. 2001. Random forests. *Machine learning* 45, 1 (2001), 5–32.

[7] Leopoldo Catania, Stefano Grassi, and Francesco Ravazzolo. 2019. Forecasting cryptocurrencies under model and parameter instability. *International Journal of Forecasting* 35, 2 (2019), 485–501. https://doi.org/10.1016/j.ijforecast.2018.09.005

[8] Tianqi Chen and Carlos Guestrin. 2016. Xgboost: A scalable tree boosting system. In *Proceedings of the 22nd acm sigkdd international conference on knowledge discovery and data mining*. 785–794.

[9] Massimo Guidolin Daniele Bianchi and Manuela Pedio. 2023. The dynamics of returns predictability in cryptocurrency markets. *The European Journal of Finance* 29, 6 (2023), 583–611. https://doi.org/10.1080/1351847X.2022.2084343 arXiv:https://doi.org/10.1080/1351847X.2022.2084343

[10] Alysha M De Livera, Rob J Hyndman, and Ralph D Snyder. 2011. Forecasting time series with complex seasonal patterns using exponential smoothing. *J. Amer. Statist. Assoc.* 106, 496 (2011), 1513–1527.

[11] J. L. Elman. 1990. Finding structure in time. *Cognitive Science* 14, 2 (1990), 179–211.

[12] Julien Herzen, Francesco Lässig, Samuele Giuliano Piazzetta, Thomas Neuer, Léo Tafti, Guillaume Raille, Tomas Van Pottelbergh, Marek Pasieka, Andrzej Skrodzki, Nicolas Huguenin, Maxime Dumonal, Jan Kościsz, Dennis Bader, Frédérick Gusset, Mounir Benheddi, Camila Williamson, Michal Kosinski, Matej Petrik, and Gaël Grosch. 2022. Darts: User-Friendly Modern Machine Learning for Time Series. *Journal of Machine Learning Research* 23, 124 (2022), 1–6. http://jmlr.org/papers/v23/21-1177.html

[13] Sepp Hochreiter and Jürgen Schmidhuber. 1997. Long short-term memory. *Neural computation* 9, 8 (1997), 1735–1780.

[14] Rob J Hyndman and George Athanasopoulos. 2021. *Forecasting: Principles and Practice (3rd ed)*. OTexts.

[15] Salim Lahmiri and Stelios Bekiros. 2019. Cryptocurrency forecasting with deep learning chaotic neural networks. *Chaos, Solitons & Fractals* 118 (2019), 35–40. https://doi.org/10.1016/j.chaos.2018.11.014

[16] Salim Lahmiri, Stelios Bekiros, and Frank Bezzina. 2022. Complexity analysis and forecasting of variations in cryptocurrency trading volume with support vector regression tuned by Bayesian optimization under different kernels: An empirical comparison from a large dataset. *Expert Systems with Applications* 209 (2022), 118349. https://doi.org/10.1016/j.eswa.2022.118349

[17] Sean McNally, Jason Roche, and Simon Caton. 2018. Predicting the Price of Bitcoin Using Machine Learning. In *2018 26th Euromicro International Conference on Parallel, Distributed and Network-based Processing (PDP)*. 339–343. https://doi.org/10.1109/PDP2018.2018.00060

[18] Ujan Mukhopadhyay, Anthony Skjellum, Oluwakemi Hambolu, Jon Oakley, Lu Yu, and Richard Brooks. 2016. A brief survey of Cryptocurrency systems. In *2016 14th Annual Conference on Privacy, Security and Trust (PST)*. 745–752. https://doi.org/10.1109/PST.2016.7906988

[19] Satoshi Nakamoto. 2008. Bitcoin: A Peer-to-Peer Electronic Cash System. https://bitcoin.org/bitcoin.pdf Accessed: 2015-07-01.

[20] Boris N Oreshkin, Dmitri Carpov, Nicolas Chapados, and Yoshua Bengio. 2019. N-BEATS: Neural basis expansion analysis for time series forecasting. In *arXiv preprint arXiv:1905.10437*.

[21] O. A. Rosso, H. A. Larrondo, M. T. Martin, A. Plastino, and M. A. Fuentes. 2007. Distinguishing Noise from Chaos. *Phys. Rev. Lett.* 99 (Oct 2007), 154102. Issue 15. https://doi.org/10.1103/PhysRevLett.99.154102

[22] Helder Sebastião and Pedro Godinho. 2021. Forecasting and trading cryptocurrencies with machine learning under changing market conditions. *Financial Innovation* 7, 1 (06 Jan 2021), 3. https://doi.org/10.1186/s40854-020-00217-x

[23] Darko Stosic, Dusan Stosic, Teresa B. Ludermir, and Tatijana Stosic. 2019. Exploring disorder and complexity in the cryptocurrency space. *Physica A: Statistical Mechanics and its Applications* 525 (2019), 548–556. https://www.sciencedirect.com/science/article/pii/S0378437119303279

[24] Ivan Svetunkov, Nikolaos Kourentzes, and John Keith Ord. 2022. Complex exponential smoothing. *Naval Research Logistics (NRL)* 69, 8 (2022), 1108–1123. https://doi.org/10.1002/nav.22074

[25] Sean J Taylor and Benjamin Letham. 2018. Forecasting at scale. *The American Statistician* 72, 1 (2018), 37–45.

[26] Junhuan Zhang, Kewei Cai, and Jiaqi Wen. 2024. A survey of deep learning applications in cryptocurrency. *iScience* 27, 1 (2024), 108509. https://doi.org/10.1016/j.isci.2023.108509

[27] Luciano Zunino, Felipe Olivares, Haroldo V. Ribeiro, and Osvaldo A. Rosso. 2022. Permutation Jensen-Shannon distance: A versatile and fast symbolic tool for complex time-series analysis. *Phys. Rev. E* 105 (Apr 2022), 045310. Issue 4. https://doi.org/10.1103/PhysRevE.105.045310

[28] Luciano Zunino, Massimiliano Zanin, Benjamin M. Tabak, Darío G. Pérez, and Osvaldo A. Rosso. 2010. Complexity-entropy causality plane: A useful approach to quantify the stock market inefficiency. *Physica A: Statistical Mechanics and its Applications* 389, 9 (2010), 1891–1901. https://doi.org/10.1016/j.physa.2010.01.007

[29] Alexandra Gabriela Ţiţan. 2015. The Efficient Market Hypothesis: Review of Specialized Literature and Empirical Research. *Procedia Economics and Finance* 32 (2015), 442–449. https://doi.org/10.1016/S2212-5671(15)01416-1 Emerging Markets Queries in Finance and Business 2014, EMQFB 2014, 24-25 October 2014, Bucharest, Romania.