# AI Agents in the Electricity Market Game with Cryptocurrency Transactions: A Post-Terminator Analysis

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### Abstract

This paper extends (Spear 2003) by replacing human agents with artificial intelligence (AI) entities that derive utility solely from electricity consumption. These AI agents must prepay for electricity using cryptocurrency and the verification of these transactions requires a fixed amount of electricity. As a result the agents must strategically allocate electricity resources between consumption and payment verification. This paper analyzes the equilibrium outcomes of such a system and discusses the implications of AI-driven energy markets.

### 1 Introduction

The electricity market has undergone significant changes due to the rise of artificial intelligence and blockchain technologies. This paper examines a theoretical model where AI agents operate within a modified version of Spear (2003). Unlike traditional models, AI agents have a singular objective: to maximize their electricity consumption while ensuring cryptocurrency transactions are verified.

The model we present here is necessarily one of post-Terminator economics, set in a future in which human beings have lost the war against the machines. In this machine-inhabited post-war economy, the only commodity having any value is electricity, which is assumed to be produced from a fixed daily influx of solar energy, subject only to the technological limits of AI-driven innovations for converting sunlight into electricity.

Since the static Shapley-Shubik model extends the standard static Arrow-Debreu model to imperfectly competitive general equilibrium environments, there is no obvious role for money or other assets. Hence, to bring the financial side of the economy into the model, we impose crypto-in-advance constraints, so that the AI agents in the model must pre-pay for their consumption of electricity using the model's (fictitious) crypto currency, which we call "bytecoins".

### 2 Model

We consider an economy consisting of AI agents, electricity producers, and a blockchain-based payment system. The fundamental changes to Spear's model include:

- AI agents replace human agents and derive utility exclusively from electricity consumption.
- Cryptocurrency serves as the sole medium for purchasing electricity, requiring prepayment before consumption.
- Each transaction must be validated via a blockchain mechanism, consuming a fixed portion of electricity.

### 2.0.1 Electricity Production

### 2.0.2 Electricity Production

We assume that the AI agents, when functioning as electricity producers, have access to a differential returns to scale electricity generation technology. assume first that there are a finite number of producers, P, and index producers by j = 1, ..., P. A producer agent produces electricity by using the consumption good as an input, together with a fixed amount of solar radiation. The production technology of a producer agent j at period t is given by Cobb-Douglas production function i.e.,  $f(\phi_i^t) = \theta(\phi_i^t)^c$  where f is a production function,  $\phi_i^t$  is the amount of consumption good used as an input at period  $t, \theta > 0$  is total factor productivity, and c is a positive constant. Notice that the production technology may exhibit constant, increasing or decreasing returns to scale<sup>1</sup> depending on the value of c. A producer agent's production capacity is fixed in the short-run  $(\overline{K})$ , and constitutes a constraint on the producer agent's optimization problem. In the long-run, on the other hand, the production capacity is an endogenous variable which will be determined by the model (K). To increase the production capacity by one unit, a producer agent needs to invest  $\rho$  units of consumption good. We will work with a sell-all aversion of the market game, so that a producer agent j's electricity offer  $q_j$  is equal to his output of electricity. Then, the producer agent j's activity vector is  $(q_j, \phi_j)$ . The collection of technically feasible activity vectors constitute a producer agent's production set. In particular, the production set of the producer agent j in the short-run is  $Y_j(\overline{K})$ 

$$Y_j(\overline{K}) = \left\{ (q_j, \phi_j) \in \mathbb{R}^{T+1} \mid 0 \le q_j^t \le \overline{K}, \text{ and } \left(\frac{1}{\theta}\right)^{1/c} \sum_{t=1}^T (q_j^t)^{1/c} \le \phi_j, \, \forall t \right\},$$

<sup>&</sup>lt;sup>1</sup>Although we refer to returns to scale, we examine only the effect of a change in the amount of the consumption good. Since this is a short-run analysis, the amount of capital is fixed.

and the production set in the long-run is  $Y_j$  where

$$Y_j = \left\{ (q_j, K, \phi_j) \in \mathbb{R}^{T+2} \mid 0 \le q_j^t \le K, \, \forall t \text{ and } \left(\frac{1}{\theta}\right)^{1/c} \sum_{t=1}^T (q_j^t)^{1/c} + \rho K \le \phi_j \right\}.$$

### 2.0.3 The Market Game

We can now formally specify the market game. The model is populated by two types of agents. Producer agents own power plants and can produce electricity. We assume there are P agents of this type and index them by j=1,...,P. AI Agents who cannot produce electricity will be called standard agents. These agents are endowed only with the blockchain technology needed to verify the value of the bytecoins traded in the model. We assume there are M agents of this type and index them by h=1,...,M. Since the demand for electricity occurs over  $T \geq 1$  periods, fully flexible pricing of demand in each period requires, in the market game setting, that transactions for power in each period and for the consumption good occur in T+1 "trading posts".

The formulation of the model below follows the Peck et al. (1992) specification in which bids are made in some unit of account (bytecoins) rather than in terms of the numeraire good. This formulation avoids some well-known problems that can occur if the availability of the numeraire ends up constraining agents' access to credit in the market. We can still make direct price comparisons of the results for the imperfectly competitive market with those for competitive markets by renormalizing the prices appropriately.

Strategies for agents h = 1, ..., M are then given by

$$S_h = \left\{ \left[ \left( \mathbf{b}_h, \xi_h \right), \left( \mathbf{0}, \omega_h \right) \right] \in \mathbb{R}_+^{2(T+1)} \right\}.$$

In keeping with the assumption that standard agents have no power production capabilities and offer all of their endowment of the consumption good on the market, h's quantity offer is just  $(\mathbf{0}, \omega_h)$ .

Agents face budget constraints on what they may bid on each of the trading posts. For agents h = 1, ..., M, the budget constraint is

$$\sum_{t=1}^{T} b_h^t + \xi_h \le \frac{B^0}{\Omega} \omega_h \tag{1}$$

where

$$B^0 = \sum_{k=1}^{M+P} b_k^0$$

and

$$\Omega = \sum_{h=1}^{M} \omega_h.$$

The constraint states that the amount that agent h can bid (in units of account) on each trading post must be less than or equal to the total amount of money

available to the agent from the sale of her endowment. For standard agents, this is given by a share of the total bid on the numeraire trading post, with the share determined by the agent's offer of endowment  $(\omega_h)$  relative to the total offer of the numeraire  $(\Omega)$ . Note that the total bid on the numeraire trading post derives both from the bids of standard agents and from those of electricity producers.

Since the aggregate bid for the numeraire includes agent h's bid, which also appears on the left-hand side of the constraint, the budget constraint can be simplified further by isolating all of agent h's bids on the left, yielding

$$\sum_{t=1}^{T} b_h^t + \frac{\Omega_{-h}}{\Omega} \xi_h \le \frac{B_{-h}^0}{\Omega} \omega_h \tag{2}$$

where

$$\Omega_{-h} = \Omega - \omega_h$$

and

$$B_{-h}^0 = B^0 - \xi_h.$$

**Producers** Producers are endowed only with the technology to produce electricity, and make offers of power on each of the electricity trading posts in the amount  $q_j^t \geq 0$  for j=1,...,P and t=1,...T. Let  $\mathbf{q}_j' = \left[q_j^1,...,q_j^t\right]$ . These agents make bids to purchase numeraire both for consumption and as inputs to production, as well as for electricity. We let  $\mathbf{b}_j' = \left[b_j^1,...,b_j^t\right]$  denote agent j's bids for electricity, and  $\xi_j$  the bid for numeraire. Producer j's strategy set is then given by

$$S_j = \left\{ \left[ \left( \mathbf{b}_j, b_j^0 \right), \left( \mathbf{q}_j, \mathbf{0} \right) \right] \in \mathbb{R}_+^{2(T+1)} \right\}$$

for j = 1, ..., P.

Producers face budget constraints on what they may bid on the electricity and numeraire trading posts. Agent j's budget constraint takes the form

$$\sum_{t=1}^{T} b_j^t + b_j^0 \le \sum_{t=1}^{T} \frac{B^t}{Q^t} q_j^t \tag{3}$$

for j = 1, ..., P, where

$$B^t = \sum_{k=1}^{M+P} b_k^t$$

and

$$Q^t = \sum_{i=1}^P q_j^t.$$

As was the case for standard consumers, producer j's budget constraint will have his bids for electricity on both the left-hand and right-hand sides of the

budget constraint, so that the constraint can be simplified by collecting the agent's own bids on the left-hand side. Doing this yields

$$\sum_{t=1}^{T} \frac{Q_{-j}^{t}}{Q^{t}} b_{j}^{t} + b_{j}^{0} \leq \sum_{t=1}^{T} \frac{B_{-j}^{t}}{Q^{t}} q_{j}^{t}$$

$$\tag{4}$$

where

$$Q_{-i}^t = Q^t - q_i^t$$

and

$$B_{-i}^t = B^t - b_i^t.$$

#### 2.0.4 Allocations

With the specifications of agents strategies given above, we now specify the allocations that agent's receive of electricity and the numeraire good. An agent's allocation of electricity in any period t will be denoted  $x_i^t$  where i denotes either a standard agent or a producer, and t=1,...,T. An agent's allocation of the numeraire good will be denoted  $x_i^0$ . With this notation, allocations are given

For h = 1, ..., M and  $t = 1, ..., T\xi$ 

$$x_h^t = \frac{b_h^t}{B^t} Q^t \tag{5}$$

and

$$x_h^0 = \frac{b_h^0}{B^0} \Omega. (6)$$

For j = 1, ..., P allocations are given by

$$x_j^t = \frac{b_j^t}{B^t} Q^t \tag{7}$$

for t = 1, ..., T, and

$$x_j^0 = \frac{b_j^0}{B^0} \Omega - \iota \cdot \varphi_j \tag{8}$$

$$\varphi_{j} = \begin{bmatrix} \varphi_{j}^{1} \\ \vdots \\ \varphi_{j}^{T} \end{bmatrix}$$

$$q_{j} = \theta (\varphi_{j}^{t})^{c} \leq K \text{ for } j = 1, ..., P$$

$$(9)$$

where  $\iota$  denotes a sum vector. If we define the price of electricity in period t by  $p^t = \frac{B^t}{Q^t}$  and of the numeraire good as  $p^0 = \frac{B^0}{\Omega}$ , then the allocations are just  $x_h^t = b_h^t/p^t$  and  $x_h^0 = b_h^0/p^0$ .

The allocations rules are quite intuititive, stating that each agent's allocation

of a commodity is determined by giving the agent the fraction of the total offer of

the good on the trading post, with the share determined by the agent's bid on the trading post as a fraction of the total bid. These rules can also be interpreted as giving the agent her bid divided by the price of the good determined on the trading post (which is given by the ratio of total bid to total quantity offered). These specifications of the allocations are standard for h=1,...,M. For producer agents, the allocation rules incorporate the constraints imposed by production. Agent j's allocation rule for electricity reflects that fact that he need not offer the full short-run capacity on the market at any point in time, although the amount he does offer must be less than capacity. The specification of j's allocation of the consumption good reflects the fact that agent j produces electricity, and hence must allocate his purchases of the consumption good between his own consumption and the input requirements for producing the output vector  $\mathbf{q}_j$ .

Finally, it is easy to verify that summing allocations over all the agents uses exactly the quantities of all goods offered on the markets, so that all markets clear.

### 2.0.5 Best Responses in the Electricity Market

Both types of agent in the model choose their bid and offer strategies as best responses to the bids and offers of other agents, that is, so as to maximize utility subject to the budget constraints, taking other agents' actions as given.

For agents h = 1, ..., M, their optimization problems are

$$\max_{(\mathbf{b}_h, \xi_h)} u_h \left( \frac{b_h^1}{B^1} Q^1, ..., \frac{b_h^T}{B^T} Q^T, \frac{\xi_h}{B^0} \Omega \right)$$

subject to

$$\sum_{t=1}^{T} b_h^t + \frac{\Omega_{-h}}{\Omega} \xi_h \le \frac{B_{-h}^0}{\Omega} \omega_h.$$

For producer agents j = 1, ..., P the optimization problem is

$$\max_{(\mathbf{b}_j, \xi_j, \mathbf{q}_j)} u_j \left( \frac{b_j^1}{B^1} Q^1, ..., \frac{b_j^T}{B^T} Q^T, \frac{b_j^0}{B^0} \Omega - \iota \cdot \varphi_j \right)$$

subject to

$$\sum_{t=1}^{T} \frac{Q_{-j}^{t}}{Q^{t}} b_{j}^{t} + b_{j}^{0} \leq \sum_{t=1}^{T} \frac{B_{-j}^{t}}{Q^{t}} q_{j}^{t}$$

and

$$q_j^t = \theta \left( \varphi_j^t \right)^c \le K \text{ for } j = 1, ..., P$$

First-order conditions for agents h = 1, ..., M are

$$u_{ht} \frac{B_{-h}^t Q^t}{(B^t)^2} - \lambda = 0 \text{ for } t = 1, ..., T$$
(10)

$$u_{h0} \frac{B_{-h}^0 \Omega}{(B^0)^2} - \lambda \frac{\Omega_{-h}}{\Omega} = 0.$$
 (11)

Those for agents j = 1, ..., P are

$$u_{jt} \frac{B_{-j}^t Q^t}{(B^t)^2} - \lambda \frac{Q_{-j}^t}{Q^t} = 0 \text{ for } t = 1, ..., T$$
 (12)

$$u_{j0} \frac{B_{-j}^0 \Omega}{(B^0)^2} - \lambda = 0 \tag{13}$$

$$u_{jt} \frac{b_{j}^{t}}{B^{t}} - u_{j0} \frac{\partial \varphi_{j}^{t}}{\partial q_{i}^{t}} + \lambda \frac{B^{t} Q_{-j}^{t}}{(Q^{t})^{2}} - \mu^{t} = 0$$
 (14)

$$u_{jt} \frac{b_j^t}{B^t} - u_{j0} \frac{1}{c\theta} \left[ \frac{q_j^t}{\theta} \right]^{\frac{1-c}{c}} + \lambda \frac{B^t Q_{-j}^t}{(Q^t)^2} - \mu^t = 0$$
 (15)

$$\mu \cdot \left[ K\iota - \mathbf{q}_j \right] = 0 \tag{16}$$

$$q_j^t = \theta \left( \varphi_j^t \right)^c \tag{17}$$

$$q_j^t = \theta \left( \varphi_j^t \right)^c \tag{17}$$

where  $\mu' = \left[\mu^1, ..., \mu^T\right]$ .

Finally, we adopt the standard definition of the Nash equilibrium as any collection of bids and offers for all agents each of which is a best response to the bids and offers of other agents.

## Optimization in the Cryptocurrency Market

Let  $E_{is}$  denote the electricity consumption of agent i, and let  $V_{is}$  represent the energy expended for payment verification in each state s. The agent's total allocation of electricity  $A_i$  in state s is then:

$$A_{is} = E_{is} + V_{is}$$

subject to budget and supply constraints.

Each AI agent solves the following optimization problem:

$$\max_{[E_i,V_i]_{s=1}^S} U(E_{is})$$

subject to the constraint:

$$A_{is} = E_{is} + V_{is} \le B_{is}$$

where  $B_{is}$  represents the prepaid budget for electricity, and the utility function U is an indirect utility derived from the Nash equilibrium outcomes examined in the previous section.

Using Lagrange multipliers, the optimal allocation satisfies:

$$\frac{dU}{dE_{is}} = \lambda_s$$

where  $\lambda_s$  is the shadow price of electricity.

# 4 Equilibrium Analysis

Given a finite set of AI agents, a Nash equilibrium exists where each agent optimally allocates energy between consumption and transaction verification. The presence of blockchain verification introduces an endogenous constraint on electricity consumption, influencing price formation and market dynamics.

### 5 Conclusion

This paper explores the interaction between AI-driven electricity consumption and cryptocurrency transactions within the modified Electricity Market Game. Future research can extend this framework by incorporating stochastic elements in transaction validation and examining competitive dynamics among AI agents.

### 6 References

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